Professor Constantin ANGHELACHE, PhD
E-mail: actincon@yahoo.com
The Bucharest University of Economic Studies
Associate Professor Mădălina-Gabriela ANGHEL, PhD
E-mail: madalinagabriela_anghel@yahoo.com
"ARTIFEX" University of Bucharest
Lecturer Ștefan Virgil IACOB, PhD
E-mail: stefaniacob79@yahoo.com
"ARTIFEX" University of Bucharest
Professor Irina Gabriela RĂDULESCU, PhD
E-mail: iradulescu@upg-ploiesti.ro
Petroleum-Gas University of Ploieşti
Lecturer Alina Gabriela BREZOI, PhD
E-mail: alina.brezoi@upg-ploiesti.ro
Petroleum-Gas University of Ploieşti

MODELS FOR DETERMINING THE EXPECTED UTILITY IN THE PLACEMENT OF ASSET PORTFOLIOS

Abstract. *In this article, the authors set out to highlight the need for an analysis based on mathematical and statistical-econometric models regarding the determination of the expected return in terms of placing assets on the capital market. It is always assumed in portfolio theory that investors usually consider only the expected return and the standard deviation of this return. We do this when comparing alternative portfolios to determine the most appropriate asset placement option.*

The utility function represents the preferences of an investor and this was done in this article by considering some short examples, which highlighted the break-even point towards which the investor is heading. The article also discusses the practical utility function because risk-averse investors consider that in any circumstance of investing in the capital market there is a certain risk that must be taken into account when choosing the possibility of placing assets.

The methodology used by the authors is a logical one, in which it compares various situations, considering a series of indicators, of variables that, put in agreement, lead to ensuring a certain trend towards which the return in the placement of certain assets evolves.

Keywords: *investors, portfolios, risk, investments, profitability, capital market, models.*

JEL Classification: C10, G11

DOI: 10.24818/18423264/56.4.22.12

1. Introduction

The analysis and establishment of the expected utility is a general theoretical activity, which involves choosing the optimal variant in conditions of uncertainty. The market is at risk and that is why a high-yield choice is a particularly important operation.

In this direction, in the model presented, the authors started from the simplification of the analysis by choosing a desired utility and, by applying mathematical relations, they highlighted the finality of this option that would lead to the expected return that the investor has in mind. Of course, there is a chance that the investor will follow the higher path to obtain a return that he anticipates.

The investor preference curve can be used to build another portfolio, one that they consider at least as attractive, and can still be other portfolios. In this sense, he is going to choose from the portfolios constituted, of course with a certain probability, the one that has the expected return.

The probability of obtaining a certain return is materialized by a mathematical function, which gives the investor the opportunity to choose, given that in all the activity of placing assets on the capital market a number of alternative portfolios are taken into account, from which investor will choose the one for which, based on the sensitivity and probability of the portfolio, to choose the one with the best results.

Sometimes the procedure used to represent investor preferences may seem arbitrary, and sometimes it is. Thus, for example, he has to take into account the various hypothetical securities that he has to consider in close accordance with what he wants. Kalayci, Can B., Okkes Ertenlice, and Mehmet Anil Akbay (2019) conducted an extensive study on deterministic models and applications for portfolio optimization.

The analysis carried out by the authors in this paper starts from the consideration of several alternative hypotheses in which a change of scale or a change of origin is involved. From this point of view, the portfolio with the highest possible value will also have the highest possible value as a final return.

There is also risk aversion in this analysis activity. In this regard, Righi, M.B., and Borenstein D. (2018) compared risk measures on the performance of optimal portfolio strategies, taking into account eleven risk measures from different classes.

A utility function represents the preferences of an investor, but in order to find the function for a certain investor, the option must be obtained when it is formed by choosing from several variants.

There are no individuals in the world of capital market investors who place without a broader or more restricted analysis of the resulting return compared to the expected result. In this regard, Zaimovic, A., Omanovic A. and Arnaut-Berilo A. (2021) point out that there is not an optimal number of shares on the market that would constitute a well-diversified portfolio and that these correlations between stocks are more the smaller the number of shares needed for a well-diversified portfolio for investors.

Next, we looked at the practical utility function because risk-averse investors are characterized by decreasing utility curves. The precise shape of any particular curve depends on how the analysis covered all the elements involved in expecting the desired yield.

When they are purchased and based on a perfect analysis, much higher returns are ensured, sometimes perhaps difficult to anticipate, in the first summary analysis performed by the investor.

Most investors prefer well-diversified portfolios, considering that their diversification ensures, in the event of a risk, its coverage by portfolio elements that avoid the risk that can be triggered.

In the article we used a series of mathematical relations on which we engraved some short and intelligible models to suggest how this analysis should be performed in order to establish the expected utility.

2. Literature review

In practice, the researchers who approached this topic went through the whole analysis procedure using mathematical and statistical-econometric models that seem to give abstract results, but in the end give meaning to the investor's desire to establish a utility that he expects when doing placements. Thus, Anagnostopoulos, K. and Georgios Mamanis G. (2010) are concerned with identifying the relationship between risk, return and number of securities in the portfolio, thus introducing into their model certain quantity and class constraints in order to limit the proportion of the portfolio. invested in assets with common characteristics and to avoid very small holdings. Baltas, I., Yannacopoulos A.N. (2019) used dynamic programming techniques to characterize the function of optimal value and solve the problem of maximizing the expected utility, providing solutions for the optimal investment decision. Baule R. (2010) conducts a study on the selection of the optimal portfolio for a small investor, considering the risk and cost of the transaction. In their paper, Byrne P. and Stephen Lee, S. (2004) take a different approach to risk measures, thus comparing portfolio holdings produced by different risk measures instead of compromising risk-return. Campbell, R., Koedijk, K. and Kofman P. (2002) consider that the method of estimating the correlation of unbiased quantum is applicable to portfolio optimization and at the same time to risk management techniques in general. This also highlights the growing correlation in extreme market conditions and its structure in multivariate yield distributions. Ferreira and Santa Clara (2011) and Hjalmarsson (2010) analyzed methods for estimating return on the capital market. A similar topic is studied by Lettau and van Nieuwerburgh (2008). Giacomini and Rossi (2010) conducted a comparative study of forecasts in unstable environments. Harvey, C.R. et al (2010) focus on the analysis of portfolio selection with high moments. Kolm et al (2014) conducted an analysis of the evolution of portfolio theory. Li J. and

Smetters K. (2011) study a number of issues regarding the choice of the optimal portfolio in the context of ensuring Social Security indexation. Matei, M., Stancu, A., Enescu, G., Geambaşu, C. (2008) addressed in their extensive paper a series of issues related to the stock market, the financial and capital market, highlighting in this regard the importance of the study of these markets and of the mathematical and statistical-econometric methods of analysis, in order to optimize the investments in order to obtain a desired profitability. Markowitz (2014) addressed issues related to the middle variant. Mba, J.C .; Ababio, K.A .; Agyei, S.K. (2022) investigates the robustness of the conventional model of optimizing the mean variance considering on the one hand the selection of the portfolio based on a behavioral decision-making theory that incorporates the statistics of the variance average and the psychology of investors, and on the other hand capturing the dependency structure. portfolio assets through copulation. Sikalo, M.; Arnaut-Berilo, A.; Zaimovic, A. (2022) compared the models for selecting the optimal portfolio taking into account the different risk measures in order to identify the periods in which they dominated. Starting from the classic Markowitz model, a series of risk measures and models for selecting the optimal portfolio have been developed. Using game theory, they presented a model for selecting the optimal portfolio based on maximum loss as a measure of risk. Simo-Kengne, Beatrice D., Kofi A. Ababio, Jules Mba, and Ur Koumba. (2018) uses the classic medium-variance model to compare the performance of the stock portfolio with different behaviors. Thomaidis, N.S., T Angelidis, T., Vassiliadis, V. And Georgios, D. (2009) impose additional constraints (limit on the maximum number of assets included in the portfolio and upper and lower limits on asset weights) on the optimization problem to identify optimal portfolios. Xu, Zuo Quan, Xun Yu Zhou, and Sheng Chao Zhuang (2019) studied the problem in which an individual's preference is of the type of rank-dependent utility and showed that an optimal contract can cover both large and small losses.

3. Methodology, data, results and discussions

- ***Use of yield and standard deviation in comparing alternative portfolios***

Portfolio theory assumes that investors only consider the expected return and the standard deviation of profitability when comparing alternative portfolios. In order to evaluate it, a more general theory of choice in conditions of uncertainty must be considered. Consider that in order to simplify the analysis of choosing the desired (expected) utility, an investor is only concerned with wealth at a certain future date. If he conforms to perfectly reasonable rules of conduct, he will be successful in his analysis.

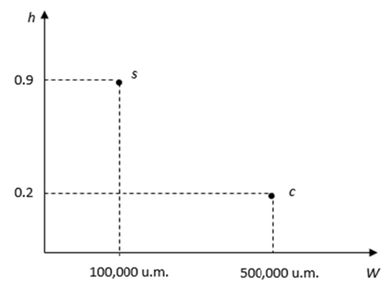

Figure 1. Alternative amounts of future wealth

The horizontal axis in Figure 1 graphically represents alternative quantities of future wealth. For convenience, we assume that any choice you can make ensures a return. These values, a lower and an upper limit, can be used to construct a series of hypothetical titles. For example, the security value of 0.5 provides 1 million u.m. with probability 0.5 or nothing with probability 0.5. The security value of 0.7 provides 1 million u.m. with probability 0.7 or nothing with probability 0.3. The security value *h* provides 1 million u.m. with probability *h* or nothing with probability (1 - *h*). Each point on the vertical axis in Figure 1 represents a hypothetical real estate value of this type. Suppose the investor (1) prefers (chooses) a security that offers 1 million u.m. with probability 0.9 or nothing with probability 0.1. This aspect is shown by point *s* (preferred security) in Figure number 1. Suppose (1) he would prefer 500,000 u.m. to the detriment of (2) a title offering 1 million u.m. with probability 0.2 or nothing with probability 0.8. This aspect is shown by point *c* (preferring a certain richness) in Figure number 1.

In Figure 2, the diagram was divided into two areas. At any point in the upper area, the security value is preferred. At any point in the lower area, certain richness is preferred. At any point on the border, the investor considers security and a certain type of gain is also attractive. Such a limit can be used to summarize the opinion of any investor, ie his preference curve. Thus, an investor's preferences can be represented by a curve that refers to h to *W*. Let (*h*', *W*') be a point on such a curve. The investor said that he is indifferent between:

1. *W* u.m. sure
2. *s* 1 million p.m. with probability *h*' or nothing with probability (1 - *h*')

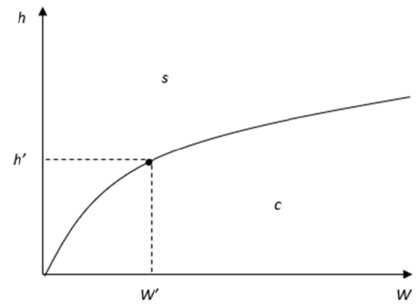

Figure 2. Preference curve

It is useful at this time to introduce a graphical representation of random developments. For example:

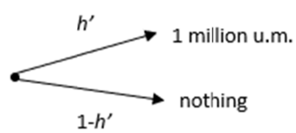

Figure 3. Random developments

This indicates that the investor is likely to follow the upper path to 1 million u.m. and a 1-*h*' probability that he will follow the lower path to nothing. Consider a portfolio with K possible results: W_1, W_2, ..., W_K, with the respective probabilities $p_1, p_2, ..., p_K$. The situation is as follows:

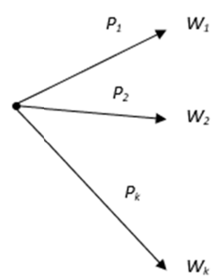

Figure 4. Portfolio with *K* results

The investor preference curve can be used to build another portfolio, one that you should consider just as attractive. If the value of h corresponds to W', i.e. (h_1, W_1), is a point on the curve of the investor's preferences, then we will also have h_2 the value of h corresponding to W_2 and so on. The investor said that he is indifferent between:

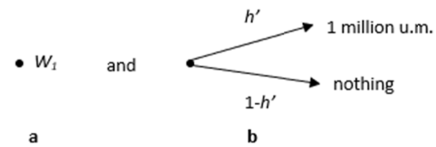

Figure 5. Indifference of the decision

The investor's decisions about a portfolio should thus be unaffected if **b** is replaced by **a**. By doing this replacement and others of the same type we get:

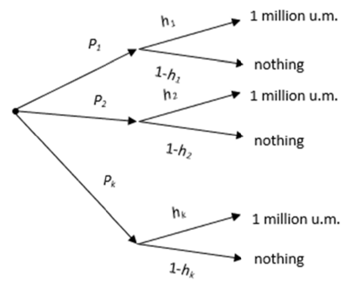

Figure 6. Evolutionary perspectives

This is a simple set of perspectives (predictions) because there are only two possible outcomes: 1 million u.m. or nothing. Probability of obtaining 1 million u.m. is given by the relation:

$$H = p_1 \cdot h_1 + p_2 \cdot h_2 + \cdots + p_k \cdot h_k \quad (1)$$

The probability of getting nothing is 1 minus this amount.

Now let's consider two portfolios. Portfolio *A* that delivers results W_1, W_2 ,... , W_K with probabilities $p_1^A, p_2^A, \dots, p_k^A$ and portfolio *B* that delivers results W_1, W_2 ,... , W_K with probabilities $p_1^B, p_2^B, \dots, p_k^B$. The investor should be indifferent between portfolio *A* and

$$H^A = p_1^A \cdot h_1 + p_2^A \cdot h_2 + \cdots + p_k^A \cdot h_k \quad (2)$$

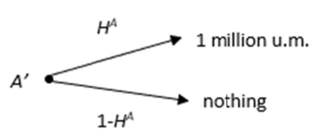

Figure 7. Random developments for portfolio *A*

It should also be indifferent between portfolio *B* and

$$H^B = p_1^B \cdot h_1 + p_2^B \cdot h_2 + \cdots + p_k^B \cdot h_k \quad (3)$$

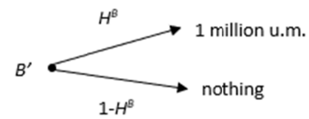

Figure 8. Random developments for portfolio *B*

It is a simple matter of highlighting the investor's feelings about *A*' and *B*'. He will prefer the one with the highest value of *H*. Suppose H^A is greater than H^B. If the investor is not reasonable, he will prefer portfolio *A* to portfolio *B*. This results from the simple transitivity of preferences:

If A is as good as *A*', which is preferred to *B*', which is as good as *B*, then *A* should be preferred to *B*.

Generalizing results:

Given any number of alternative portfolios, an investor will choose the one for which:

$$H = \sum_{k=1}^{K} p_k \cdot h_k \quad (4)$$

where *H* = a measure of the sensitivity of a portfolio

p_k = the probability that a portfolio will provide the investor with wealth W_K

h_k = the value of h associated with wealth W_K along the investor preference curve

K = the number of possible results

The procedure used to represent the investor's preferences may seem arbitrary and, in a way, it is. Various hypothetical securities could have been used. E.g.:

1. The hypothetical security value *h* could provide 2 million u.m. with probability *h*, or nothing with probability (1 - *h*)
2. The hypothetical security value h could provide 2 million u.m. with probability *h* or 1 million p.m. with probability (1 - *h*)

The first alternative would involve a change of scale, and the second would involve a change of origin.

Consider a function that expresses u at h as follows:

$$u_k = c_o + c_s \cdot h_k \quad (5)$$

Where c_o = constant (indicating origin)

c_s = positive constant (indicating the scale)

u_k = value of u corresponding to h_k (value of *h*)

$$EU = \sum_{k=1}^{K} p_k \cdot u_k \quad (6)$$

Substituting u_k in equation (6) it follows:

$$EU = \sum_{k=1}^{K} p_k \cdot (c_o + c_s \cdot h_k) = c_o \cdot \sum_{k=1}^{K} p_k + c_s \cdot \sum_{k=1}^{K} p_k \cdot h_k = c_o + c_s \cdot H \quad (7)$$

We find that the portfolio with the highest possible value of *H* will also have the highest possible value of the *EU*.

A preference curve can be obtained by arbitrarily defining hypothetical securities and then asking the investor to express their preferences for such securities over certain yields. A curve that reports u to W can be obtained using any desired c_o origin and c_s scale. The initial preference curve is shape:

$$u_k = 0 + 1h_k \tag{8}$$

Compared to the historical precedent, its value will be called the investor's utility. A curve that reports u to W will be called a utility function or utility curve. Usually, an investor will choose the portfolio that maximizes, respectively:

$$EU_p = \sum_{k=1}^{K} p_k \cdot u_k \tag{9}$$

where EUp = expected utility of the portfolio

p_k = the probability that the portfolio will provide the investor with an expected return

u_k = utility associated with the return along the investor's utility curve

This is called the maximum expected utility.

Its derivation deserves to be re-portrayed. It is assumed that the investor behaves in ways that seem natural. In particular, it was assumed that:

- the investor was able to identify alternatives that were neither better nor worse than each possible amount of gain (the preference curve existed);
- its portfolio preferences would not be affected if monetary outcomes were replaced by equally common market games (i.e. those on its preference curve);
- his opinion on each portfolio depends only on the probabilities of the different results;
- his preferences were transitive (i.e. if he chose X over Y and was indifferent between Y and Z, then he would choose X over Z).

The preferences of any investor with a similar vision can be represented by (1) a function that links its profitability to profit plus (2) the statement that it will always choose the portfolio with the highest expected utility.

Normally they have to behave in quite different ways. It could be called unjustified or even irrational.

- ***Risk aversion***

A utility function is the preferences of an investor. To find a position for a particular investor, one must get his vision when faced with various variants. Some characteristics of an investor regarding the utility function can be predicted.

Utility curves have similarities and differences. First, they are tilted up. This must be the case if both W and h are goods. All other things being equal, it can be assumed that an investor prefers a higher return. It can be assumed that the investor prefers with a higher probability to receive 1 million u.m. with a lower risk.

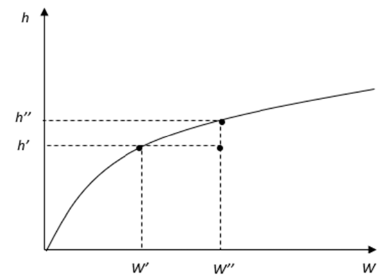

Figure 9. Two portfolios relative to the preference curve

Consider Figure 9. It follows that the investor will prefer *W*'' to *W*' because the former is larger. He will also prefer *W*'' to *h*' because *W*' and *h*' are equally safe. Thus, the point (*W*'', *h*') must be below the curve of the investor's preference (utility). If an investor is at risk, his profit curve will tilt upward in some way, increasing at a declining rate. So far, risk aversion has been characterized in terms of attitudes towards the standard deviation of the rate of return. A risk averse investor considers σ_p, other things equal, he prefers less to more. Such an investor will only hold a risky portfolio if it offers a sufficiently higher expected return than a risk-free one. We will consider the portfolio *p*. For an investor, a risk-free portfolio would be just as desirable. Such a portfolio would certainly provide some value. The investor's solution regarding the *p* portfolio can be usefully summarized with this value:

$CE_p = \text{equivalent certainty of portfolio growth } p$

The investor in question would certainly obtain the value of *CE*p, as well as the set of perspectives associated with the portfolio *p*. The current prospects of a portfolio can be considered with an expected value, respectively:

$EW_p = \text{expected portfolio growth } p$

$$EW_p = \sum_{k=1}^{K} p_k \cdot W_k \tag{10}$$

A risk-free portfolio certainly offers some growth. Both the guaranteed equivalent value and the expected increase must be equal. Thus, for a risk-free portfolio, it turns out:

$$CE_p = EW_p \tag{11}$$

As we analyze in the case of risky portfolios. Investors who are indifferent to risk would rate it as risk-free. Risk-averse investors would consider such portfolios to be less desirable than comparable risk-free portfolios. Investors who prefer risk would consider it desirable. These differences serve to define the three possible attitudes towards risk. For a risky portfolio:

$CE_p < EW_p$ for risk-averse investors
$CE_p = EW_p$ for investors indifferent to risk
$CE_p > EW_p$ for investors who prefer risk

This is in line with the definition of risk aversion. We will consider two portfolios with the same gain. One risky and the other risk-free. An investor with a risk aversion would attribute to the risky portfolio a smaller increase equivalent to certainty, or would be willing to maintain it only if it were cheaper. But, if it were cheaper, the risky portfolio would offer a higher expected rate of return. So, a risk averse investor wants a higher expected return from a risky portfolio than a risk-free one.

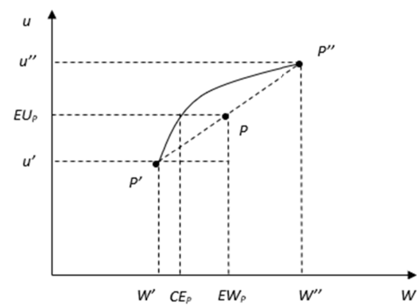

Figure 10. Risk aversion

A risk averse investor considers a risky portfolio less desirable than a risk-free portfolio with the same expected increase. This attitude defines risk aversion. It also implies that the investor's profit curve increases at a decreasing rate.

We will consider Figure number 10. W' and W'' are two possible values of growth, and u' and u'' are the corresponding amounts of utility for an investor. Points P and P'' are on its utility curve.

Let's imagine a portfolio that offers a 50% - 50% chance of getting either W' or W''. Graphically, this is a summary of:

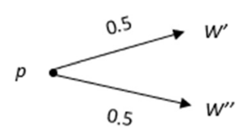

Figure 11. Equal opportunities portfolio

The expected utility of the portfolio is given by the relationship:

$$EU_p = 0.5 \cdot u' + 0.5 \cdot u'' \qquad (12)$$

The expected increase is:

$$EW_p = 0.5 \cdot W' + 0.5 \cdot W'' \qquad (13)$$

The point P, in Figure 10 represents these two values, which must be on the straight line connecting the points P' and P''.

The certainly equivalent gain of the P portfolio is defined as a certain amount (value) that the investor considers as achievable as the portfolio, having the same expected utility. Clearly, the certainly equivalent increase in the *P* portfolio is the value of W for which $u = EU$p (i.e. the point on the investor's utility curve at which the utility is equal to EUp). A risk averse investor considers that the certainly equivalent gain of a risky portfolio is less than the expected gain. This implies that the utility curve must be to the left of point *P,* in Figure 10. In other words, the curve must be to the left of the line *P*'*P*'' at all points other than *P*' and *P*''. This can be demonstrated by considering portfolios with different chances of receiving W' and W''. The values of W' and W'' used in our example are arbitrary. Any other values could have been chosen, leading to the same general conclusion, namely, risk aversion is a utility curve that increases at a decreasing rate. No market law requires all investors to be risk-averse. A single investor might be upset about the risk of decisions that could result in a range of outcomes, but in fact they prefer the risk to decisions that could have negative results in another period.

- ***The quadratic utility functions***

Risk-averse investors are characterized by rising profit curves. The precise shape of any particular curve will depend on the attitude of the investor. In the continuation of the analysis we will resort to a simplifying hypothesis. A model is consistent with choices based exclusively on expected return and standard deviation of profitability, respectively the assumption that utility is a quadratic function of gain. Let's imagine that a person who chose to invest a sum W_o. Each possible amount in his portfolio later can be linked to a certain rate of return on investment. Thus, the utility can be made depending on the increase or the rate of return, as shown in Figure number 12. For this purpose, it is desirable to use the rate of return. Thus, we consider a quadratic utility function of the form:

$$u = a + b \cdot r - c \cdot r^2 \qquad (14)$$

where u = utility
r = rate of return
a = constant
b = positive constant
c = positive constant

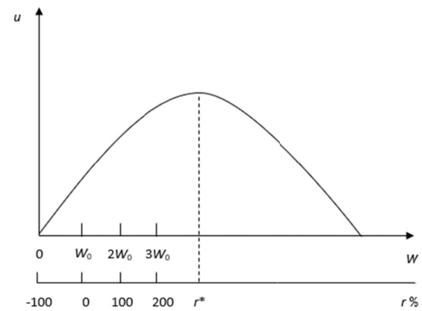

Figure 12. Quadratic utility and rate of return

The general appearance of such a function is shown in Figure 12, and the exact shape of the curve will depend on the values of parameters *a*, *b*, and *c*.

This type of function reaches a maximum of a certain value of *r*, denoted *r**. Beyond this point, profitability decreases as the rate of return increases. This aspect becomes unacceptable. Such a curve should never be used for decisions with results over *r*'. We must assume that within a relevant range of the rate of return, the real profit curve of an investor can be appropriately approximated by a square curve of shape (14).

Figure 13 provides an illustration and indicates that an investor's utility curve is shaped (14).

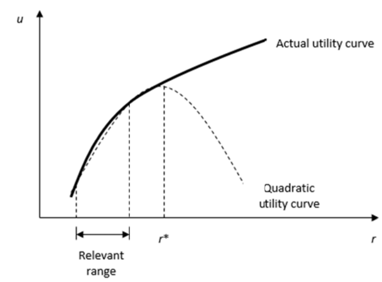

Figure 13. Normal, quadratic utility curve and optimal interval

If we consider a portfolio it can offer *K* rates of return $(r_1, r_2, \dots, r_k)$ with probabilities $(p_1, p_2, \dots, p_k)$ and u_k the rate of return associated with the utility r_k, then the utility curve is shaped:

$$u_k = a + b \cdot r_k - c \cdot r_k^2 \tag{15}$$

Thus, the expected utility of a portfolio will be:

$$EU_p = \sum_{k=1}^{K} p_k \cdot u_k = \sum_{k=1}^{K} p_k \cdot \left(a + b \cdot r_k - c \cdot r_k^2\right) = a \cdot \sum_{k=1}^{K} p_k + b \cdot \sum_{k=1}^{K} p_k \cdot r_k - c \cdot \sum_{k=1}^{K} p_k \cdot r_k^2 \tag{16}$$

where: $\sum_{k=1}^{K} p_k = 1$ și $\sum_{k=1}^{K} p_k \cdot r_k = E_p$

The expected utility will be:

$$EU_p = a + b \cdot E_p - c \cdot E_p^2 - c \cdot \sigma_p^2 \tag{17}$$

The maximum expected utility implies that an investor will be indifferent between portfolios with the same expected utility. Considering all portfolios with a certain expected utility (*EU**) each must have an expected return and a standard deviation of profitability in accordance with the equation:

$$EU^* = a + b \cdot E_p - c \cdot E_p^2 - c \cdot \sigma_p^2 \tag{18}$$

Values E_p and σ_p who meet this requirement are on the same indifference curve. Targeting E_p and σ_p as variables, this is the equation of an indifference curve.

Equation (18) can be written like this: $c \cdot E_p^2 - b \cdot E_p + c \cdot \sigma_p^2 = a - EU^*$. Dividing by *c* we get:

$$E_p^2 - \frac{b}{c} \cdot E_p + \sigma_p^2 = \frac{a - EU^*}{c}$$

Add $\frac{b^2}{4 \cdot c^2}$ on both sides of equality and we get:

$$E_p^2 - \frac{b}{c} \cdot E_p + \frac{b^2}{4 \cdot c^2} + \sigma_p^2 = \frac{a - EU^*}{c} + \frac{b^2}{4 \cdot c^2}$$

Rewrite:

$$\left(E_p - \frac{b}{2c}\right)^2 + \sigma_p^2 = \frac{a - EU^*}{c} + \frac{b^2}{4 \cdot c^2} \quad (19)$$

This is the equation of a circle centered at $E_p = \frac{b}{2c}$ and $\sigma_p = 0$.

If we consider equation (14), respectively $u = a + b \cdot r - c \cdot r^2$ and we derive according to *u* and *r*, we obtain:

$$\frac{du}{dr} = b - 2cr \quad (20)$$

This must be zero for *r**, which implies:

$$b - 2cr^* = 0 => r^* = \frac{b}{2c} \quad (21)$$

So, the indifference curve is a circle centered at $E_p = r^*$ and $\sigma_p = 0$.

In Figure 14, *CML*' represents the available alternatives and the preferred point *P*'. The investor assumes a risk equal to *σ*' for a return expected by E_p'. We will now assume that the expected return on each security, including risk-free securities, increases by 5%. The investor is richer in real terms. In Figure 15, the new line of the capital market is *CML*", and the point *P*" is now preferred and involves a risk equal to σ_p and an expected return of E_p".

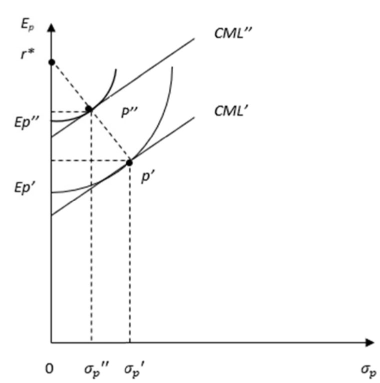

Figure 14. Variants available depending on the degree of risk

Risky securities are said to be normal goods, things bought in larger quantities as the investor gets richer. But simple graphics exclude such behavior as long as the indifference curves are concentric circles centered on the vertical axis. The argument went on. Thus, the minimum assumptions about choice in conditions of uncertainty imply a behavior compatible with maximizing the expected utility. The analysis with only E_p and σ_p is consistent with a quadratic utility function. But a quadratic utility function involves indifference curves that are represented as concentric circles centered on the vertical axis. Also, such curves cannot represent the preferences of some investors in a completely satisfactory manner. Each point in a diagram E_p, σ_p can be interpreted as representing a specific distribution of probability, i.e. that of the rate of return of a well-diversified portfolio. Any investor should be able to classify all such possibilities. In other words, one can construct a set of indifference curves that reflect an investor's preferences between such alternatives. And curves can have any shape, in the sense that they do not have to be circular. We can say that it is necessary to use the initial justification to assume that investors choose from the portfolios only on the basis of the expected return and the standard deviation of the return. It is a convenient simplifying hypothesis. Along with other hypotheses, it leads to important and useful implications.

3. Conclusions

The study of this article on the analysis and determination of the expected utility shows a number of conclusions that must be taken into account when an investor wants to place assets on the capital market.

A first requirement is that the portfolio theory be known by the investor and that he use mathematical and statistical-econometric relations (models) in his analysis, which should highlight the risk perspective in the placement of the portfolio. Also be able to perform a comparative analysis between a risky and a risk-free portfolio placed under the same conditions, but knowing the risk elements that may arise and that may be known, diminished and may sometimes be eliminated. In the analysis performed, the investor must take into account the structure of the portfolio and the overall risk or rather the risk to which each constituent element of the portfolio is exposed. In this way, he will be able to make a general assessment of how the structure of the portfolio may lead to hedging risks, elements subject to this perspective, but at the same time increase the yield in the event that risks do not arise due to market changes or are easily offset by the use of a utility function that involves appropriate measures. Risk aversion is a very interesting element and it should be analyzed in terms of attitudes towards the standard deviation of a return. Of course, any portfolio involves a certain deviation depending on the risks, lower or higher, but there is also a standard deviation that must be established by the analyst so that, if this happens, he can think of a substantial increase in risk.

Another conclusion is that a risk-free portfolio ensures a certain growth for sure, but which is guaranteed with certainty and, in this context, the prospect of an expected stability is fulfilled. But risky portfolios can ensure, if well analyzed and the results interpreted, a much more substantial increase in earnings, respectively in the final return. The gain equivalent to the certainty of a portfolio is defined as a certain amount that the investor considers equally reasonable in the case of the established portfolio. Certainly, the increase equivalent to the certainty of the portfolio is a value that can be determined on a point of a utility curve constructed following the analysis. Risk aversion considers that a gain equivalent to the certainty of a risky portfolio is lower than the expected gain. This implies that the utility curve is at a well-defined point so that it is the boundary between additional earnings over the guaranteed one. Suppose that certain amounts are invested and these included in the portfolio can give a return on investment only if the analysis carried out took into account all market conditions, all parameters that may arise.

Another conclusion is that the maximum expected utility implies that the investor will make a profit, no matter which portfolio he chooses, if the results from the analysis were similar. Thus, for example, on the inference curve, the combinations thought by the investor must go through all the points that require concretizations, through the use of mathematical and statistical-econometric methods.

Finally, we can specify that for the investors of major portfolios, the activity must have a careful analysis in advance, so as to establish the practical utility compared to the expected utility.

REFERENCES

[1] **Anagnostopoulos, K., Mamanis G. (2010),** ***A Portfolio Optimization Model with Three Objectives and Discrete Variables.*** *Computers & Operations Research*, 37: 1285-97;

[2] **Baltas, I., Yannacopoulos A.N. (2019),** ***Portfolio Management in a Stochastic Factor Model under the Existence of Private Information***. *IMA Journal of Management Mathematics*, 30(1): 77-103;

[3] **Baule, R. (2010),** ***Optimal Portfolio Selection for the Small Investor Considering Risk and Transaction Costs.*** *OR Spectrum*, 32(1): 61-76;

[4] **Byrne P., Lee, S. (2004),** ***Different Risk Measures: Different Portfolio Compositions?***. *Journal of Property Investment & Finance,* 22: 501-11;

[5] **Campbell, R., Koedijk, K., Kofman P. (2002),** ***Increased Correlation in Bear Markets.*** *Financial Analysts Journal*, 58: 87-94;

[6] **Ferreira, M.A., Santa-Clara, P. (2011),** ***Forecasting Stock market Returns: The Sum of the Parts is More than the Whole***. *Journal of Financial Economics*, 100, 514-537;

[7] **Giacomini, R., Rossi, B. (2010),** ***Forecast Comparisons in Unstable Environments. Journal of Applied Econometrics***; *John Wiley & Sons, Ltd.,* 25(4): 595-620;

[8] **Harvey, C.R. et al. (2010),** ***Portfolio Selection with Higher Moments.*** *Quantitative Finance*, 10(5), 469-485;

[9] **Hjalmarsson, E. (2010),** ***Predicting Global Stock Returns.*** *Journal of Financial and Quantitative Analysis*, *Cambridge University Press*, 45(1): 49-80;

[10] **Kalayci, C.B., Ertenlice, O., Akbay, M.A. (2019),** ***A Comprehensive Review of Deterministic Models and Applications for Mean-Variance Portfolio Optimization*****.** *Expert Systems with Applications*, 125: 345-68;
[11] **Kolm, P.N. et al. (2014),** ***60 Years of Portfolio Optimization: Practical Challenges and Current Trends***. *European Journal of Operational Research*, 234: 356-371;
[12] **Lettau, M., van Nieuwerburgh, S. (2008),** ***Reconciling the Return Predictability Evidence***. *Review of Financial Studies*, 21(4): 1607-1652;
[13] **Li, J., Smetters, K. (2011),** ***Optimal Portfolio Choice with Wage-indexed Social Security***, *Working Paper 17025*;
[14] **Matei, M., Stancu, A., Enescu, G., Geambaşu, C. (2008),** ***Burse de mărfuri şi valori*****.** *Universitatea din Ploieşti Publishing;*
[15] **Markowitz, H. (2014),** ***Mean-variance Approximations to Expected Utility***. *European Journal of Operational Research*, 234: 346-355;
[16] **Mba, J.C., Ababio, K.A., Agyei, S.K. (2022),** ***Markowitz Mean-Variance Portfolio Selection and Optimization under a Behavioral Spectacle: New Empirical Evidence***. *International Journal of Financial Studies*, 10, 28;
[17] **Righi, M.B., Borenstein D. (2018),** ***A Simulation Comparison of Risk Measures for Portfolio Optimization***. *Finance Research Letters*, 24: 105-12;
[18] **Sikalo, M., Arnaut-Berilo, A., Zaimovic, A. (2022),** ***Efficient Asset Allocation: Application of Game Theory-Based Model for Superior Performance***. *International Journal of Financial Studies*, 10(1), 20;
[19] **Simo-Kengne, Beatrice D., Kofi A. Ababio, Jules Mba, Ur. Koumba (2018),** ***Behavioral Portfolio Selection and Optimization: An Application to International Stocks.*** *Financial Markets and Portfolio Management*, 32: 311-28;
[20] **Thomaidis, N.S., T Angelidis, T., Vassiliadis, V., Dounias, G. (2009),** ***Active Portfolio Management with Cardinality Constraints: An Application of Particle Swarm Optimization*****.** *New Mathematics and Natural Computation*, 5(3): 535-555;
[21] **Xu, Z. Q., Xun, Y.Z., Sheng, C.Z. (2019),** ***Optimal Insurance under Rank-Dependent Utility and Incentive Compatibility*****.** *Mathematical Finance*, 29: 659-92;
[22] **Zaimovic, A., Omanovic A., Arnaut-Berilo A. (2021),** ***How Many Stocks Are Sufficient for Equity Portfolio Diversification? A Review of the Literature.*** *Journal of Risk and Financial Management*, 14: 551.